Matematisk-fysiske Meddelelser udgivet af<br>Det Kongelige Danske Videnskabernes Selskab Bind 32, nr. 3<br>Mat. Fys. Medd. Dan. Vid. Selsk. 32, no. 3 (1960)

# ON THE STRUCTURE OF CESIUM HEXAHALOGENO-PLUMBATES (II) 

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København 1960
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## Synopsis

Crystalline compounds of composition $\mathrm{Cs}_{4} \mathrm{PbX}_{6}$, in which $\mathrm{X}=\mathrm{Cl}, \mathrm{Br}$, or I may be prepared either from aqueous solutions or by fusing CsX and $\mathrm{PbX} \mathrm{X}_{2}$ together. $X$-ray investigations show that the crystals are rhombohedral and belong to space group No. $167 R \overline{3} c$. The hexagonal unit cells have dimensions $a=13.18_{7} \mathrm{~A}, c=16.64_{1} \mathrm{~A}$ for $\mathrm{Cs}_{4} \mathrm{PbCl}_{6} ; a=13.73_{2} \mathrm{~A}, c=17.32_{4} \mathrm{~A}$ for $\mathrm{Cs}_{4} \mathrm{PbBr}_{6}$; $a=14.52_{8} \mathrm{~A}, c=18.31_{3} \mathrm{~A}$ for $\mathrm{Cs}_{4} \mathrm{PbI}_{6}$, and contain six "molecules". The molar volumes of $\mathrm{Cs}_{4} \mathrm{PbX}_{6}$ are larger than the sum of the molar volumes of CsX and $\mathrm{PbX}_{2}$ by c. 6 cc . per gram-atom halogen. - Atomic parameters can be obtained from fairly well resolved Patterson- and electron-projections on (1210). The $\mathrm{Pb}-$ and X -atoms form isolated, slightly distorted $\mathrm{PbX}_{6}$-octahedra; the PbCl -distances are $2.93 \AA$ and the PbBr -distances are 3.12 A , i. e. in both cases a little longer than in the $\mathrm{CsPbX}_{3}$-compound, but shorter than the sum of the ionic radii. The agreement between calculated and observed structure factors is not so good as wanted. This may in part be due to a small delocalization around their average positions of the Cs-atoms having an irregular 8-coordination.

## Introduction

In the course of a systematic study of the structure of the plumbo-halogen complex ions it became of interest also to examine the crystalline compounds with the general formula $\mathrm{Cs}_{4} \mathrm{PbX}_{6}$. The interpretation of the Guinier diagram of $\mathrm{CsPbCl}_{3}$ had already made it necessary to investigate the powder diagram of $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}{ }^{1}$ and from this start we have continued work on structures of this type. When it had progressed so far that most of the facts mentioned in the present article could materialize it was understood from a footnote in a paper by G. Bergerhof and O. Schmitz-Dumont ${ }^{2}$ that these authors had also been engaged on the same problem. However, they have not, apparently, published any results of their investigations, and we have therefore taken the opportunity to give an account of our own work on these compounds as they are closely related to the previously investigated structure of type $\mathrm{CsPbX}_{3}$.

## Preparation and some general properties

The crystalline compounds $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}$ and $\mathrm{Cs}_{4} \mathrm{PbBr}_{6}$ can be made in two ways: (1) By dissolving $\mathrm{PbX}_{2}$ in a boiling aqueous solution which is almost saturated with CsX at room temperature, filtering while still hot and then leave to slow cooling. ${ }^{3}$ (2) By fusing CsX and $\mathrm{PbX}_{2}$ together in the correct stoichiometric proportion. - As for $\mathrm{Cs}_{4} \mathrm{PbI}_{6}$ this compound had not been prepared before; it was obtained (unintentionally) by boiling $\mathrm{PbBr}_{2}$ in an aqueous solution nearly saturated with a fifty-fifty mixture of CsBr and CsI. It does not seem possible to obtain it in a pure state by melting $\mathrm{PbI}_{2}$ and CsI together in the stoichiometric proportion: The powder diagrams of such products besides lines of $\mathrm{Cs}_{4} \mathrm{PbI}_{6}$ invariably also show the presence of

[^0]CsI-lines. Apart from this the identity of compounds obtained in the different ways was shown from their $X$-ray powder diagrams.

The crystals grown from solutions were colourless and their most prominent faces were parallelogram-shaped with angles $72^{\circ}-60^{\circ}$ and $108^{\circ}-120^{\circ}$. Under the polarizing microscope the Cl - and Br -compounds showed diagonal extinction with $\alpha$ bisecting the acute angles. The refractive indices were high, but the birefringence low. On the assumption that the crystals were hexagonal (proved from the $X$-ray work) it could be inferred from conoscopic examination that they were uniaxially positive in all three cases.

The density was determined on single crystals of $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}$ by flotation in Clerici solutions of different densities to be $3.70 \pm 0.07 \mathrm{~g} / \mathrm{cm}^{3}$.

No changes of the crystals could be observed on heating them to about $320^{\circ} \mathrm{C}$.

## Space group and unit cell dimensions

$X$-ray investigations on powders of $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}, \mathrm{Cs}_{4} \mathrm{PbBr}_{6}$, and $\mathrm{Cs}_{4} \mathrm{PbI}_{6}$, and on single crystals of the first two were made along the same lines as in the previous paper (1).

The Weissenberg diagrams taken about twofold axes of the single crystals could be indexed on a hexagonal lattice. Reflections of type hkil were only observed for $-h+k+l=3 n$, and $h \bar{h} 0 l$ only for $l=2 n$, so that the space group can be either No. $167 R \overline{3} c$ or No. $161 R 3 c$, but as no piezoelectric effect could be detected by a dynamical method, ${ }^{4}$ the former seems most likely. The Guinier diagrams could be indexed by application of the general quadratic form:

$$
\begin{equation*}
\sin ^{2} \vartheta=\frac{\lambda^{2}}{3 a^{2}}\left(h^{2}+k^{2}+h k\right)+\frac{\lambda^{2}}{4 c^{2}} l^{2}=\frac{a^{* 2}}{4} \cdot s+\frac{c^{* 2}}{4} l^{2} \tag{1}
\end{equation*}
$$

The result of this together with the unit cell dimensions derived from the coefficients to $s$ and $l^{2}$ are given in Table $1(\lambda=1.5418 \AA)$.

The number of molecules per hexagonal unit cell of $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}$ is found from the unit cell volume, density of the crystals and molecular weight to be 5.9 , i. e. 6. As it is beyond doubt that $\mathrm{Cs}_{4} \mathrm{PbBr}_{6}$ and $\mathrm{Cs}_{4} \mathrm{PbI}_{6}$ are both isomorphous with $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}$, it is concluded that they also have 6 molecules per hexagonal unit cell (or 2 molecules per rhombohedral unit cell).
${ }^{4}$ W. Frank, Unpublished work.

Table 1. Comparison of observed and calculated $\sin ^{2} \vartheta$-values for $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}, \mathrm{Cs}_{4} \mathrm{PbBr}_{6}$ and $\mathrm{Cs}_{4} \mathrm{PbI}_{6}$

|  | $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}$ |  |  | $\mathrm{Cs}_{4} \mathrm{PbBr}_{6}$ |  |  | $\mathrm{Cs}_{4} \mathrm{PbI}_{6}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Indices hkil | Intensity estimated | $\begin{gathered} 10^{4} \\ \sin ^{2} \vartheta_{\text {obs. }} \end{gathered}$ | $\begin{gathered} 10^{4} \\ \sin ^{2} \vartheta_{\text {calc. }} \end{gathered}$ | Intensity estimated | $\begin{gathered} 10^{4} \\ \sin ^{2} \vartheta_{\text {obs }} \end{gathered}$ | $\begin{gathered} 10^{4} \\ \sin ^{2} \vartheta_{\text {calc. }} \end{gathered}$ | Intensity estimated | $\begin{gathered} 10^{4} \\ \sin ^{2} \vartheta_{\text {obs }} \end{gathered}$ | $\begin{gathered} 10^{4} \\ \sin ^{2} \vartheta_{\text {calc }} \end{gathered}$ |
| $\overline{1} 012$ | m-s | 0132 | 0132 | m | 0120 | 0121 | m | 0109 | 0109 |
| $11 \overline{2} 0$ | m-s | 0137 | 0137 | m-(w) | 0126 | 0126 | m-w | 0113 | 0113 |
| $11 \overline{2} 3$ | S | 0330 | 0330 | m | 0304 | 0304 | vw? | 0270 | 0273 |
| $21 \overline{3} 1$ | (v) w | 0341 | 0340 |  | . . |  |  | . | . |
| $\overline{2} \overline{1} 32$ | . . | . . | . . | vw | 0373 | 0373 | (v) W | 0335 | 0334 |
| $10 \overline{1} 4$ | w-m | 0389 | 0388 |  | . . |  |  | . | . |
| $30 \overline{3} 0$ | (v) s | 0411 | 0410 | (m)-s | 0378 | 0378 | $\mathrm{m}(-\mathrm{w})$ | 0339 | 0338 |
| $\overline{2} 024$ | m-s | 0526 | 0525 | m-s | 0485 | 0485 | m | 0435 | 0434 |
| $22 \overline{4} 0$ | vw | 0547 | 0547 |  | . . | . . | . | . . | . . |
| $\overline{3} \overline{1} 41$ | m | 0614 | 0614 | m-s | 0566 | 0566 | m-(s) | 0506 | 0506 |
| $30 \overline{3} 3$ | . | - | . | . . | . $\cdot$ | . . | vw? | 0496 | 0498 |
| $21 \overline{3} 4$ | (m)-s | 0662 | 0662 | s | 0611 | 0611 | m-s | 0547 | 0547 |
| $31 \overline{4} 2$ | (w)-m | 0679 | 0678 | w-m | 0626 | 0625 | m | 0560 | 0560 |
| $22 \overline{4} 3$ | (m)-s | 0740 | 0740 | s | 0683 | 0682 | (m)s | 0610 | 0610 |
| 0006 | w | 0772 | 0772 | w-m | 0712 | 0713 | w | 0639 | 0639 |
| $\overline{4} 042$ | vw | 0815 | 0815 |  |  |  |  |  |  |
| $\overline{2} \overline{1} 35$ | W | 0856 | 0856 | vw | 0789 | 0789 | vw | 0706 | 0707 |
| $32 \overline{5} 1$ | vw? | 0888 | 0888 |  |  |  |  |  |  |
| $11 \overline{2} 6$ | w-m | 0909 | 0910 | w-m | 0838 | 0839 | . | . $\cdot$ | $\cdots$ |
| $\overline{3} \overline{1} 44$ | w-m | 0936 | 0935 | w | 0862 | 0863 | vw | 0772 | 0772 |
| $41 \overline{5} 0$ | w-m | 0957 | 0957 | m-w | 0882 | 0882 | w | 0788 | 0789 |
| $40 \overline{4} 4$ | w | 1072 | 1072 | vw | 0989 | 0989 | vw? | 0885 | 0886 |
| $31 \overline{4} 5$ | w-(m) | 1130 | 1128 | w | 1042 | 1041 | w | 0932 | 0932 |
| 4153 | w-m | 1150 | 1150 | v-w | 1061 | 1060 | . . |  |  |
| $30 \overline{3} 6$ | w | 1183 | 1182 | . | . | . $\cdot$ | . | . | . |
| $32 \overline{5} 4$ | s-m | 1209 | 1209 | m-s | 1116 | 1115 | m | 0996 | 0997 |
| $33 \overline{6} 0$ | m | 1229 | 1230 | w | 1135 | 1134.5 | (v) W | 1012 | 1013.5 |
| $\overline{4} \overline{2} 61$ | w | 1298 | 1298 | . | . . | . . | . . |  |  |
| $22 \overline{4} 6$ | w-m | 1319 | 1320 | w | 1216 | 1217 | w | 1090 | 1089 |
| $42 \overline{6} 2$ | w | 1362 | 1362 | . | . | . |  | . | . |
| $\overline{1} 018$ | W | 1419 | 1418 | w | 1310 | 1309 | w | 1173 | 1174 |
| $51 \overline{6} 1$ | w | 1434 | 1434 | (v) w | 1323 | 1323 | w | 1179 | 1182 |
| 5054 | W | 1481 | 1482 | vw | 1366 | 1367 | . | . . | . . |
| $20 \overline{2} 8$ | w | 1556 | 1555 | vw? | 1433 | 1435 | vw? | c. 1284 | 1286 |
| $\overline{4} \overline{2} 64$ | w | 1621 | 1619 | . . | . . | . . | . . | . . | . . |
| $\overline{3} \overline{1} 47$ | w-m | 1645 | 1644 | . . | . | $\cdots$ | . | . | . |
| $60 \overline{6} 0$ | . . | . . | . . | w | 1512 | 1513 | m-w | 1353 | 1351 |
| $\overline{2} \overline{1} 38$ | w | 1692 | 1692 | vw | 1562 | 1561 | vw | 1399 | 1399 |
| 415 | w | 1729 | 1730 | . | . | $\cdots$ | $\cdots$ | $\cdots$ | . |

(to be continued)

Table 1 (continued).

|  | $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}$ |  |  | $\mathrm{Cs}_{4} \mathrm{PbBr}_{6}$ |  |  | $\mathrm{Cs}_{4} \mathrm{~Pb} \mathrm{I}_{6}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Indices hkil | Intensity estimated | $\begin{gathered} 10^{4} \\ \sin ^{2} \vartheta_{\text {obs }} \end{gathered}$ | $\begin{gathered} 10^{4} \\ \sin ^{2} \vartheta_{\text {calc }} \end{gathered}$ | Intensity estimated | $\begin{gathered} 10^{4} \\ \sin ^{2} \vartheta_{\text {obs. }} \end{gathered}$ | $\begin{gathered} 10^{4} \\ \sin ^{2} \vartheta_{\text {calc. }} \end{gathered}$ | Intensity estimated | $\begin{gathered} 10^{4} \\ \sin ^{2} \vartheta_{\text {obs }} \end{gathered}$ | $\begin{gathered} 10^{4} \\ \sin ^{2} \vartheta_{\text {calc }} \end{gathered}$ |
| $51 \overline{6} 4$ | v-w | 1755 | 1756 |  | . | . |  |  |  |
| $52 \overline{7} 0$ |  | 1776 | 1777 | vw | 1638 | 1639 |  | 1460 f | 1464 |
| - 4372 |  |  | 1772 |  |  |  | W | 1460 | 1460 |
| $42 \overline{6} 5$ | vw? | 1814 | 1812 |  | . |  | S | 1495 | 1494 |
| $11 \overline{2} 9$ | (v)-w | 1875 | 1875 |  | - | - |  |  |  |
| 万5165 | vw | 1949 | 1950 | vw | 1796 | 1798 | vw | $160{ }_{6}$ | 1608 |
| $\overline{6} \overline{1} 71$ | (v)-w | 1985 | 1981 |  |  | . | vw | 1631 | 1632 |
| $61 \overline{7} 2$ | (v)-w | 2044 | 2046 |  | 1880 f | 1886 | . | . . | . . |
| $43 \overline{7} 4$ | v-w | 2028 | 2029 | vw | 1880 \{ | 1872 |  |  |  |
| $\overline{4} 048$ | vw | 2102 | 2102 |  |  | . . |  |  |  |
| $10 \overline{1} 10$ | 1 vw | 2193 | 2192 | . | $\ldots$ |  |  | 181 | 1813 |
| $\overline{5} 057$ | vw | 2193 | 2190 |  |  |  | ¢ $v w$ ? | 181- | 1807 |
| $\overline{3} \overline{2} 58$ | m | 2241 | 2239 | vw | 2067 | 2065 | vw | 1848 | 1849 |
| $22 \overline{4} 9$ | w | 2283 | 2285 | vw | 2111 | 2108 | vw? | 1887 | 1888 |
| $61 \overline{7} 5$ | . | . . | . | m-s | 2298 | 2302 | . . | . . | . . |
| $50 \overline{5} 8$ | . . | . | . | vw? | 2318 | 2317 | vw | 2075 | $2072 \cdot 5$ |
| $\overline{5} \overline{3} 84$ |  |  |  |  |  |  | ) w-m |  |  |
| $70 \overline{7} 4$ |  |  |  |  |  | . | $\int^{\text {w }}$-m | 2122 | 2122.5 |
| Unit cell: | $a=13.18$ | $8_{7} \AA ; c=$ $\text { ol: } 2506$ | $16.64_{1} \AA$ | $a=13.73$ | $\begin{aligned} & { }_{2} \AA ; c= \\ & \text { ol: } 2829 \end{aligned}$ | $\begin{aligned} & 17.32_{4} \AA \\ & \AA^{3} \end{aligned}$ | $a=14.5^{\circ}$ <br> Vo | $\begin{aligned} & 28 \AA ; c= \\ & \mathrm{ol}: 3447 \end{aligned}$ | $18.31_{3} \AA$ |

## Volume increments

It would seem interesting to compare the molar volumes of the compounds $\mathrm{Cs}_{4} \mathrm{PbX}_{6}$ and $\mathrm{CsPbX}_{3}$ obtained from $X$-ray analysis with the (crystal) molar volumes of CsX and $\mathrm{PbX}_{2}$. The latter have been calculated from the molecular weights and the densities of the simple, crystalline compounds, but practically the same values are obtained from $X$-ray determination of the unit cell dimensions. ${ }^{5}$ Table 3 shows the result of this comparison. The "calculated" molar volumes are those to be expected from the molar volumes of CsX and $\mathrm{PbX}_{2}$ if the rule about additivity of molar volumes were valid.

In all cases of formation of these compounds from simpler ones there is thus seen to be an increase in volume. It amounts to roughly 6 ce per

[^1]Table 2. Molar volumes and volume increments per gr.-atom halogen

| Compound |  | $\mathrm{X}=\mathrm{Cl}$ | $\mathrm{X}=\mathrm{Br}$ | $\mathrm{X}=\mathrm{J}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \mathrm{CsX} \\ & \mathrm{PbX}_{2} \end{aligned}$ |  | $\begin{aligned} & 42.4 \mathrm{cc} \\ & 47.6- \end{aligned}$ | $\begin{aligned} & 48.0 \text { cc } \\ & 55.0 \end{aligned}$ | $\begin{aligned} & 57.6 \text { cc } \\ & 74.8- \end{aligned}$ |
|  |  |  |  |  |
| $\mathrm{CsPbX}{ }_{3}$ | $\left\{\begin{array}{l}\text { Calculated....... } \\ \text { Observed . . . . . } \\ \text { Increment per } \mathrm{X}\end{array}\right.$ | 90.0 | 103.0 | 132.4 - |
|  |  | 106.0 - | 120.5 | $141.9 \mathrm{cc} \quad 134.1 \mathrm{cc}$ |
|  |  | 5.3 | 5.8 - | $3.2 \mathrm{cc} \quad 0.6 \mathrm{cc}$ |
| $\mathrm{Cs}_{4} \mathrm{PbX}_{6}$ | Calculated | 217.2 - | 247.0 - | 305.2 cc |
|  | Observed | 251.5 | 283.9 | 346.0 - |
|  | Increment per X | 5.7 | 6.2 | 6.8 - |

gramatom of halogen except for $\mathrm{CsPbI}_{3}$ in which it is practically negligible for the orthorhombic form.

As the structure of the latter compound-and also the structures of $\mathrm{PbX}_{2}$-strongly indicate polarization of the halogen ions by the Pb -ions it seems reasonable to guess that the volume increments are connected with a change of the $\mathrm{Pb}-\mathrm{X}$-bonding-presumably an increase of ionic character - . In the perovskite-like structures of $\mathrm{CsPbX}_{3}$ the less economic packing of the ions makes it possible for the cæsium and the halogen ions to "choose", among several close-lying potential minima (1).

If this is the case also in crystals of the $\mathrm{Cs}_{4} \mathrm{PbX}_{6}$-compounds, one might expect difficulties in localizing the Cs - and X -atoms accurately by $X$-ray methods.

## Atomic arrangements

With six molecules $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}$ in a unit cell belonging to space group No. 167 most of the atomic positions should be fixed by symmetry and only four parameters have to be determined. ${ }^{6}$

Patterson projections on the plane (1 $\overline{2} 10$ ) were calculated for both $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}$ and $\mathrm{Cs}_{4} \mathrm{PbBr}_{6}$ from Weissenberg diagrams taken about a twofold axis (crystal dimensions were about $75 \mu$ ). The results are shown in figs. 1 and 2. The maxima at $\left(0, \frac{1}{4}\right)$ are required by space group No. 167 if $6 \mathrm{Cs}^{\prime}$ are placed in the special positions " $a$ " and 6 Pb in the special positions " $b$ " (see Ref. 6 p. 275). The remaining $18 \mathrm{Cs}^{\prime}$-atoms are then likely to be in special positions " $e$ ", and on this assumption the peaks marked Cs-Cs

[^2]

Fig. 1 and 2. Patterson projections on ( $1 \overline{2} 10$ ) of $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}$ (left) and $\mathrm{Cs}_{4} \mathrm{PbBr}_{6}$ (right). Contour lines have been drawn at the (relative) levels: 0 (broken line) $50,100,150,200,300,350$.
and $\mathrm{Pb}-\mathrm{Cs}$ provide the only missing Cs-parameter: $x=0.37$. Having fixed the Pb - and Cs-atoms we are able to determine the signs for most of the structure factors $\mathrm{F}(h 0 \bar{h} l)$ of $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}$, and an electron projection on ( $1 \overline{2} 10$ ) was calculated. This is fairly well resolved (fig. 3). It is possible to locate the Cl-atoms with some certainty and thus to determine the still remaining three Cl-parameters. Structure factors calculated from the atomic parameters obtained in this way and from atomic scattering factors corrected for dispersion effects (1) show a fair agreement with the observed ones. However, attempts to refine the structures either by trial and error or from difference synthesis of $\varrho_{\text {obs. }}-\varrho_{\text {calc. }}$. were not successful (even less so for $\mathrm{Cs}_{4} \mathrm{PbBr}_{6}$ !). The differences between strong and weak reflections obtained experimentally are much less pronounced than indicated by the calculated structure factors. This might be due to extinction, which is almost certain to be of importance in these crystals. Although no immediate absorption effects can be seen on the Weissenberg diagrams such ones may also be responsible for the lack of


Fig. 3. Electron projection on $(1 \overline{2} 10)$ of $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}$. Contour lines have been drawn as in figs. 1 and 2.
agreement: The shape of the crystals did not even approximate to something having cylindrical symmetry. (It would have been desirable to use much smaller crystals, but it was found very difficult to mount them along a body diagonal without using a micromanipulator).

However, the difference syntheses indicate that a better agreement between observed and calculated structure factors presumably could be obtained by reducing the atomic scattering factors for the $\mathrm{Cs}^{\prime \prime}$-atoms in one way or another. This means formally introduction of a special, anisotropic temperature factor for these atoms, and physically a small delocalization of them around the average positions. In fact, it is possible in this way to get a somewhat better agreement in certain regions, but, as will be understood from the above, our measured intensities do not seem accurate enough and the whole procedure has been left out here. It is interesting that considerations of this kind achieved a definite improvement in the case of the perovskitelike structures of $\mathrm{CsPbX}_{3}$ (1).

Although a completely satisfactory solution of the problem has not been

Table 3. Comparison of calculated and observed F -values for $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}$

| Indices $h k l$ | $\mathrm{F}_{\text {calc }}$ | \| $\mathrm{F}_{\text {obs. }} \mid$ | Indices hkl | $\mathrm{F}_{\text {calc }}$ | Fobs. | Indices hkl | $\mathrm{F}_{\text {calc }}$ | F ${ }_{\text {obs }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 006 | - 180 | 118 | $70 \overline{14}$ | 19 |  | 211 | 24 |  |
| 0012 | 220 | 182 | $70 \overline{8}$ | 80 | 112 | 214 | 102 | 104 |
| 0018 | - 80 | 115 | $70 \overline{2}$ | 17 | 54 | 217 | - 11 |  |
| $10 \overline{20}$ | 74 | 108 | 704 | 88 | 65 | 2110 | 29 |  |
| $10 \overline{14}$ | 12 |  | 7010 | 22 |  | 2113 | 4 |  |
| $10 \overline{8}$ | 91 | 87 | 7016 | 52 |  | 2116 | 60 | 79 |
| $10 \overline{2}$ | 58 | 65 | $80 \overline{16}$ | 164 | 126 | 2119 | - 1 |  |
| 104 | 66 | 52 | $80 \overline{10}$ | $-72$ | 71 | 312 | 78 | 83 |
| 1010 | 64 | 71 | $80 \overline{4}$ | 170 | 160 | 315 | $-66$ | 58 |
| 1016 | 35 |  | 802 | $-51$ | 60 | 318 | 43 | 52 |
| $20 \overline{16}$ | 104 | 180 | 808 | 126 | 123 | 3111 | 42 |  |
| $20 \overline{10}$ | - 16 |  | 8014 | $-25$ |  | 3114 | 58 |  |
| $20 \overline{4}$ | 147 | 125 | $90 \overline{12}$ | 39 |  | 3117 | $-26$ |  |
| 202 | $-2$ |  | $90 \overline{6}$ | 42 | 65 | 3120 | 27 | 52 |
| 208 | 117 | 121 | 900 | 63 | 72 | 410 | 82 | 86 |
| 2014 | 3 |  | 906 | 18 |  | 413 | 61 | 52 |
| 2020 | 75 | 97 | 9012 | 78 | 86 | 416 | 44 | 45 |
| $30 \overline{18}$ | - 36 |  | $100 \overline{14}$ | 10 |  | 419 | -41 |  |
| $30 \overline{12}$ | 142 | 166 | $100 \overline{8}$ | 74 | 93 | 4112 | 55 | 63 |
| $30 \overline{6}$ | - 53 | 58 | $100 \overline{2}$ | 7 |  | 4115 | 29 |  |
| 300 | 208 | 132 | 1004 | 86 | 108 | 4118 | 37 |  |
| 306 | - 79 | 71 | 10010 | 2 |  | 511 | 79 | 70 |
| 3012 | 172 | 186 | $110 \overline{10}$ | - 76 |  | 514 | 73 | 63 |
| 3018 | - 70 | 46 | $110 \overline{4}$ | 172 | 123 | 517 | -64 | 43 |
| $40 \overline{20}$ | 48 | 76 | 1102 | -91 |  | 5110 | 38 |  |
| $40 \overline{14}$ | 28 |  | 1108 | 166 | 134 | 5113 | 44 |  |
| $40 \overline{8}$ | 74 | 97 | $120 \overline{12}$ | 36 |  | 5116 | 50 | 58 |
| $40 \overline{2}$ | 46 | 67 | $120 \overline{6}$ | 46 |  | 5119 | $-28$ |  |
| 404 | 72 | 82 | 1200 | 31 | 39 | 612 | 50 | 81 |
| 4010 | 53 | 44 | $1206$ | 50 |  | 615 | $-97$ | 88 |
| 4016 | 35 |  | 12012 | 28 |  | 618 | 41 | 54 |
| $50 \overline{16}$ | 104 | 110 |  |  |  | 6111 | 85 | 88 |
| $50 \overline{10}$ | - 28 |  |  |  |  | 6114 | 48 |  |
| $50 \overline{4}$ | 163 | 152 | 012 |  | 36 | 710 | 52 | 58 |
| 502 | - 58 | 80 | 018 | 92 | 86 | 713 | 15 |  |
| $508$ | 169 | 152 | 0114 | 11 |  | 716 | 40 |  |
| 5014 | -63 | 60 | 0120 | 73 | 86 | 719 | $-12$ |  |
| $5020$ | 134 | 97 | 110 | 54 | 63 | 7112 | 61 | 50 |
| $60 \overline{18}$ | $-26$ |  | 113 | 90 | 65 | 7115 | 10 |  |
| $60 \overline{12}$ | 129 | 125 | 116 | 82 | 71 | 811 | 28 |  |
| $60 \overline{6}$ | -54 | 50 | 119 | - 80 | 65 | 814 | 72 | 81 |
| 600 | 179 | 177 | 1112 | 37 |  | 817 | $-26$ |  |
| 606 | $-54$ | 50 | 1115 | 66 | 63 | 8110 | 30 |  |
| $6012$ | $130$ | 126 | $1118$ | $51$ |  | $8113$ | 24 |  |
| 6018 | $-28$ |  | 1121 | -61 | 52 | 8116 | 41 | 56 |

Table 4. Comparison of calculated and observed F -values for $\mathrm{Cs}_{4} \mathrm{PbBr}_{6}$

| Indices $h k l$ | $\mathrm{F}_{\text {calc }}$. | \| $\mathrm{F}_{\text {obs. }}$ \| | Indices $h k l$ | $\mathrm{F}_{\text {calc }}$ | Fobs. | Indices $h k l$ | $\mathrm{F}_{\text {calc }}$. | \| Fobs. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 006 | $-251$ | 126 | $40 \overline{14}$ | 11 |  | $80 \overline{16}$ | 219 | 184 |
| 0012 | 246 | 210 | $40 \overline{8}$ | 97 | 117 | $80 \overline{10}$ | - 120 | 93 |
| 0018 | -52 |  | $40 \overline{2}$ | 37 | 42 | $80 \overline{4}$ | 172 | 153 |
| $10 \overline{20}$ | 106 | 130 | 404 | 62 | 82 | 802 | $-10$ |  |
| $10 \overline{14}$ | -22 | 33 | 4010 | 77 | 81 | 808 | 72 | 61 |
| $10 \overline{8}$ | 113 | 88 | 4016 | 12 | 20 | 8014 | 16 |  |
| $10 \overline{2}$ | 70 | 81 | $50 \overline{16}$ | 79 | 92 | $90 \overline{12}$ | 10 |  |
| 104 | 22 |  | $50 \overline{10}$ | 20 | 32 | $90 \overline{6}$ | 76 | 100 |
| 1010 | 109 | 122 | $50 \overline{4}$ | 118 | 139 | 900 | 47 |  |
| 1016 | 10 | 42 | 502 | -46 | 55 | 906 | 36 | 19 |
| 1022 | 46 | 46 | 508 | 200 | 178 | 9012 | 101 | 94 |
| $20 \overline{16}$ | 126 | 161 | 5014 | -49 | 83 | $100 \overline{14}$ |  |  |
| $20 \overline{10}$ | -37 | 30 | 5020 | 201 | 169 | $100 \overline{8}$ | 76 | 93 |
| $20 \overline{4}$ | 154 | 158 | $60 \overline{18}$ | -9 | 20 | $100 \overline{2}$ | 12 |  |
| 202 | 10 |  | $60 \overline{12}$ | 129 | 127 | 1004 | 85 | 78 |
| 208 | 98 | 176 | $60 \overline{6}$ | $-72$ | 90 | 10010 | -1 |  |
| 2014 | 17 |  | 600 | 214 | 189 | $110 \overline{10}$ | -63 |  |
| 2020 | 73 | 101 | 606 | - 72 | 59 | $110 \overline{4}$ | 172 | 107 |
| $30 \overline{18}$ | -25 |  | 6012 | 140 | 116 | 1102 | -98 | 36 |
|  | 110 | 111 | 6018 | 25 |  | 1108 | 174 | 101 |
| $30 \overline{6}$ | -9 | 19 | $70 \overline{14}$ | -2 |  | 11014 | -74 |  |
| 300 | 179 | 188 | $70 \overline{8}$ | 115 | 118 | $120 \overline{12}$ | 17 | 49 |
| 306 | -78 | 55 | $70 \overline{2}$ | -73 |  | $120 \overline{6}$ | 70 |  |
| 3012 | 197 | 251 | 704 | 127 | 85 | 1200 | 15 | 24 |
| 3018 | -98 | 72 | 7010 | 12 | 39 | 1206 | 54 | 35 |
| $40 \overline{20}$ | 59 | 69 | 7016 | 61 | 33 | 12012 | 42 | 14 |

obtained, it seems unlikely that the atomic positions can be very wrong. They are listed in Table 5 and interatomic distances calculated on the basis of this in Table 6. The latter compare quite reasonably with values obtained from the structures of $\mathrm{CsPbCl}_{3}$ and $\mathrm{CsPbBr}_{3}$, and, as there, the Pb -atoms are surrounded by 6 halogen atoms, which form a nearly regular octahedron (symmetry: $4 \overline{3} \mathrm{~m}$ ). In the present structures the octahedron is compressed along a trigonal axis-viz. the one parallel to the c-axis of the crystals-so that the symmetry becomes $\overline{3} m$ only. The XPbX-angle, where the two X -atoms have the same z -coordinate, is also given in Table 6.

From what has been said above, it is clear that these results must be taken with a certain reservation, and the uncertainty as to the obtained interatomic distances may easily be $0.10 \AA$, thus making the deviation of these angles from $90^{\circ}$ illusory.

Six X -atoms belonging to two different $\mathrm{PbX}_{6}$-octahedra form trigonal prisms (symmetry 32 m ) with the $\mathrm{Cs}^{\prime}$-atoms as centres, the $\mathrm{Cs}^{\prime}-\mathrm{X}$ distances

Table 5. Atomic positions in $\mathrm{Cs}_{4} \mathrm{PbX}_{6}$
$(0,0,0) ; \quad\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right) ; \quad\left(\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)+$
6 Pb in $\overline{3}: \quad(0,0,0) ; \quad\left(0,0, \frac{1}{2}\right)$
$6 \mathrm{Cs}^{\prime}$ in $32:\left(0,0, \frac{1}{4}\right) ; \quad\left(0,0, \frac{3}{4}\right)$
$18 \mathrm{Cs}^{\prime \prime}$ in $2:\left(x, 0, \frac{1}{4}\right) ; \quad\left(0, x, \frac{1}{4}\right) ; \quad\left(\bar{x}, \bar{x}, \frac{1}{4}\right) ; \quad\left(\bar{x}, 0, \frac{3}{4}\right) ; \quad\left(0, \bar{x} \frac{3}{4}\right) ; \quad\left(x, x, \frac{3}{4}\right)$
For $\mathrm{Cs}_{4} \mathrm{PbCl}_{6} x=0.370$ and for $\mathrm{Cs}_{4} \mathrm{PbBr}_{6} x=0.372$
36 X in $1: \quad(x, y, z) ; \quad(\bar{y}, x-y, z) ; \quad(y-x, \bar{x}, z) ; \quad\left(\bar{y}, \bar{x}, \frac{1}{2}+z\right) ; \quad\left(x, x-y, \frac{1}{2}+z\right) ; \quad\left(y-x, y, \frac{1}{2}+z\right) ;$ + inverse positions.
For Cl in $\mathrm{Cs}_{4} \mathrm{PbCl}_{6}: x=0.040 \quad y=0.164 \quad z=0.095$
For Br in $\mathrm{Cs}_{4} \mathrm{PbBr}_{6}: x=0.035 \quad y=0.170 \quad z=0.099$

Table 6. Interatomic distances between nearest neighbours in $\mathrm{Cs}_{4} \mathrm{PbX}_{6}$

| Distance (frequency) | From this study | From Pauling's ionic radii | ```From Goldschmidt's ionic radii``` | $\begin{aligned} & \text { From } \mathrm{CsPbX}_{3} \\ & \text { in Ref. 1. } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \mathrm{Pb}-\mathrm{Cl}(6) \\ \mathrm{Cs}^{\prime}-\mathrm{Cl}(6) \\ \mathrm{Cs}^{\prime \prime}-\mathrm{Cl}(2) \\ \mathrm{Cs}^{\prime \prime}-\mathrm{Cl}(2) \\ \mathrm{Cs}^{\prime \prime}-\mathrm{Cl}(2) \\ \mathrm{Cs}^{\prime \prime}-\mathrm{Cl}(2) \\ \mathrm{Cl}^{-\mathrm{Cl}} . . \\ \Varangle \mathrm{ClPbCl} \end{gathered}$ | $\begin{gathered} 2.93 \AA \\ 3.57- \\ 3.59- \\ 3.85- \\ 3.43- \\ 3.43- \\ 4.01-4.28 \AA \\ 93.6^{\circ} \end{gathered}$ | $\begin{aligned} & 3.02 \AA \\ & 3.50- \end{aligned}$ | $\begin{aligned} & 3.13 \AA \\ & 3.48- \end{aligned}$ | $\begin{gathered} 2.86 \AA \\ (3.68)- \end{gathered}$ |
|  | $\begin{gathered} 3.12 \AA \\ 3.69- \\ 3.67- \\ 3.93- \\ 3.65- \\ 3.60- \\ 4.31-4.52 \AA \\ 92.8^{\circ} \end{gathered}$ | $\begin{aligned} & 3.16 \AA \\ & 3.64- \end{aligned}$ | $\begin{aligned} & 3.28 \AA \\ & 3.63- \end{aligned}$ | $\begin{gathered} 2.99 \AA \\ (3.85)- \end{gathered}$ |

not being very different from the values expected from ionic radii. ${ }^{7}$ - The remaining $18 \mathrm{Cs}^{\prime \prime}$-atoms in positions of rather low symmetry are each

[^3]surrounded by 8 X -atoms, $2 \times 3$ from two $\mathrm{PbX}_{6}$-octahedra and $2 \times 1$ from two others, but here the $\mathrm{Cs}^{\prime \prime}$ - X -distances vary considerably, and it is conceivable that a disordered structure may result.

The crystals of $\mathrm{Cs}_{4} \mathrm{PbX}_{6}$ thus contain isolated $\mathrm{PbX}_{6}{ }^{----}$-octahedra linked together by the positive $\mathrm{Cs}^{+}$-ions.

## Discussion

Apart from the supposition about disorder our conclusions concerning the structures of the $\mathrm{Cs}_{4} \mathrm{PbX}_{6}$-compounds are in complete agreement with the findings of G. Bergerhof and O. Schmitz-Dumont in the isomorphous $\mathrm{K}_{4} \mathrm{CdCl}_{6}$-crystals. This again illustrates the similarity between divalent $\mathrm{Pb}^{++}$ and $\mathrm{Cd}^{++}$previously demonstrated in the case of the $\mathrm{CsPbI}_{3}$ and $\mathrm{MeCdCl}_{3}$ compounds. ${ }^{8}$ It is interesting that the compounds $\mathrm{Cs}_{4} \mathrm{PbX}_{6}$ with isolated $\mathrm{PbX}_{6}$-octahedra are precipitated from concentrated solutions of CsX , whereas from less concentrated solutions crystals are obtained in which such octahedra share X -atoms, e.g. in the perovskite-like structures of $\mathrm{CsPbX} 3_{3}$, or in the one-dimensional polynuclear complex ion $\left(\mathrm{PbI}_{3}{ }^{-}\right)_{n}$ in $\mathrm{CsPbI}_{3}$ (8). It thus seems likely that the concentrated solutions mainly contain isolated highly symmetrical $\mathrm{PbX}_{6}{ }^{----}$-ions, while in less concentrated ones bi-, tri- - - or polynuclear complex ions built up from such units may be prevailing. This may add to our understanding of the problem why in concentrated salt solutions the behaviour of some electrolytes approaches to that of electrolytes in extremely dilute solutions.

## Acknowledgements

I am very much indebted to Professor, dr. phil. A. Tovborg Jensen for his interest in this work and for his encouragement at critical points of the investigations. Thanks are also due to Mr. F. Ammentorp-Schmidt for carrying out most of the calculations-also those which were not successful.

[^4]


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